Commutativity Reasoning for Automated Distributed Coordination

> Mohsen Lesani University of California, Riverside





 $Req_1, Req_2, Req_3, \ldots$

 Req_1 , Req_3 , Req_2 , ...

 Req_1 , Req_3 , Req_2 , ...







Consistency vs. Responsiveness and Availability



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Consistency vs. Responsiveness and Availability



Confusing Weak Consistency Notions

🖄 stack overflow	About Products For Teams Q Search
Home	Strong Consistency in Cassandra
PUBLIC	
S Questions	
Tags	According to datastax article, strong consistency can be guaranteed if, R + W > N where R is the consistency level of read operations W is the consistency level of write operations N is the
Users	7 number of replicas
COLLECTIVES 0	What does strong consistency mean here? Does it mean that 'every time' a query's response is
C Explore Collectives	given from the database, the response will 'always' be the last updated value? If conditions of strong consistency is maintained in caseandre, then, are there no concrise where the data
FIND A JOB	returned might be inconsistent? In short, does strong consistency mean 100% consistency?
Jobs	Edit 1
Companies	Adding some additional material regarding some scenarios where Cassandra might not be
TEAMS	consistent even when R+W>RF
Stack Overflow for	1. Write fails with Quorum CL
Teams – Collaborate and share knowledge with a private group.	2. Cassandra's eventual consistency

Confusing Weak Consistency Notions

istack overflow	About Products For Teams Q Search
Home PUBLIC	Data consistency in DynamoDB
© Questions Tags Users	 I want to use DynamoDB for a large scale service which would be accessed by many users within a second. I want to know how correct would be the read data from DynamoDb which provides "Eventual Consistent" reads.
COLLECTIVES ① Collectives FIND A JOB	 This link <u>http://docs.aws.amazon.com/amazondynamodb/latest/developerguide</u> <u>/APISummary.html</u> says "Consistency across all copies of the data is usually reached within a second". I haven't tried testing SQL DBs for such highly accessed databases, but the service provided by DynamoDB doesn't seem to be better at least.
Jobs Companies	The strongly consistent read is costly and may take more time, so I prefer the normal reads. I necessary I'll have to check for strongly consistent read.
TEAMS	I am little bit afraid of the "Eventual" word. Has anyone seen such a scenario where Dynamol
Stack Overflow for Teams – Collaborate	?

Consistency and Integrity

• Bank Account. Integrity: Non-negative balance.

- Bank Account. Integrity: Non-negative balance.
- What users need is integrity and Consistency is just a means to Integrity.

Class

Integrity Property

Synthesis of replicated objects that preserve integrity and convergence and minimize coordination





Well-coordination:

Synchronization between conflicting Causality between dependent

Theorem:

Well-coordination is sufficient for integrity and convergence





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      register(s) := \lambda \langle ss, cs, es \rangle.
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 $\begin{aligned} \mathcal{C}(\sigma_2, c_2) &= \\ g(\sigma_2) \land \\ \mathcal{I}(\sigma_2) \end{aligned}$







Conflict Dependency

1



1



1



1



1



1



1



1



1



1



1



1



1



1



\mathcal{S} -commute



S-commute



S-commute



S-commute



\mathcal{S} -commute



\mathcal{S} -commute














2) Permissible-Conflict

$\mathcal{P}\text{-conflict}$



2) Permissible-Conflict

$\mathcal{P}\text{-conflict}$















































\mathcal{S} -commute

\mathcal{S} -commute



\mathcal{S} -commute

$\mathcal{P}\text{-}\mathrm{concur}$

Concur S-commute $\land \mathcal{P}$ -concur

\mathcal{S} -commute

$\mathcal{P}\text{-}\mathrm{concur}$

Concur S-commute $\land \mathcal{P}$ -concur

Conflict

 \neg Concur

$\mathcal{S} ext{-commute}$

	r	а	e	d	q
r	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
а	\checkmark	\checkmark	\checkmark	×	\checkmark
e	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
d	\checkmark	×	\checkmark	\checkmark	\checkmark
q	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark

 $\mathcal{P} ext{-concur}$

	r	а	e	d	q
r	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
а	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
e	\checkmark	\checkmark	\checkmark	Х	\checkmark
d	\checkmark	\checkmark	×	\checkmark	\checkmark
q	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark

Concur S-commute $\land \mathcal{P}$ -concur

	r	а	e	d	q
r	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
а	\checkmark	\checkmark	\checkmark	×	\checkmark
e	\checkmark	\checkmark	\checkmark	×	\checkmark
d	\checkmark	X	X	\checkmark	\checkmark
q	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark

Conflict

 \neg Concur

\mathcal{S} -commute

	r	а	e	d	q
r	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
а	\checkmark	\checkmark	\checkmark	×	\checkmark
e	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
d	\checkmark	×	\checkmark	\checkmark	\checkmark
q	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark

 $\mathcal{P} ext{-concur}$

	r	а	e	d	q
r	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
а	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
e	\checkmark	\checkmark	\checkmark	Х	\checkmark
d	\checkmark	\checkmark	×	\checkmark	\checkmark
q	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark

Concur S-commute $\land \mathcal{P}$ -concur

d а e r q \checkmark \checkmark \checkmark r \checkmark \checkmark Х а \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark Х \checkmark e d \checkmark \checkmark \checkmark \times Х \checkmark \checkmark \checkmark \checkmark \checkmark q

Conflict

 \neg Concur



















 \mathcal{I} -Sufficient
















Independent

I-Sufficient $\lor \mathcal{P}$ -L-commute

Independent I-Sufficient $\lor \mathcal{P}$ -L-commute

Dependent

 \neg Independent

Independent

 $\mathcal{I}\text{-Sufficient} ~\lor~ \mathcal{P}\text{-L-commute}$

	r	а	e	d	q
r	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
a	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
e	×	X	\checkmark	\checkmark	\checkmark
d	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
q	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark

Dependent

 \neg Independent



- Well-coordination
 - Locally permissible
 - Conflict-synchronizing
 - Dependency-preserving
- Theorem: Well-coordination is sufficient for integrity and convergence.







 rep_2









 $\mathcal{P} ext{-L-Commutativity}$





 \mathcal{P} -R-Commutativity





 $\mathcal{S} ext{-commute}$



 $\mathcal{S} ext{-commute}$









 $\mathcal{S} ext{-commute}$





 $\mathcal{S} ext{-commute}$





 $\mathcal{S} ext{-commute}$
















































Maximal Cliques



Maximal Cliques

Experimental Results



Asymmetric Synchronization



Asymmetric Synchronization



Asymmetric Synchronization



Guarantees

- Convergence
- Integrity
- Recency?









Reservation	Movie	
User ID: Integer Movie ID: Integer	Movie ID: Integer Available Space: Integ	er







 $\mathsf{book}(\langle u,m\rangle)$

 $\mathsf{cancelBook}(\langle u,m \rangle)$

 $\operatorname{offScreen}(m)$

 $\mathsf{specialReserve}(\langle m,n\rangle)$

 $\mathsf{increaseSpace}(\langle m,n\rangle)$

















querySpace $(m) := 3 \lambda \langle rs, ms \rangle$. queryReservations $(u) := 4 \lambda \langle rs, ms \rangle$. querySpaces $(u) := 6 \lambda \langle rs, ms \rangle$.

$$[querySpace(m)] = [3]\lambda \langle rs, ms \rangle. \quad \dots$$

queryReservations $(u) := 4 \lambda \langle rs, ms \rangle$.

querySpaces
$$(u) := 6 \lambda \langle rs, ms \rangle$$
.
querySpace(m) :=
$$3 \lambda \langle rs, ms \rangle$$
.
queryReservations(u) := $4 \lambda \langle rs, ms \rangle$.
querySpaces(u) := $6 \lambda \langle rs, ms \rangle$.

querySpace $(m) := 3 \lambda \langle rs, ms \rangle$. queryReservations $(u) := 4 \lambda \langle rs, ms \rangle$. querySpaces $(u) := 6 \lambda \langle rs, ms \rangle$.



request issued



request issued



request issued



request issued



request issued





request issued





request issued





request issued







request return

^جے synchronization





request return

^جے synchronization





request return

^جے synchronization





request return

^جتر synchronization















1. All-state-commutativity







3. Invariant-sufficiency



3. Invariant-sufficiency



4. Let-P-R-commutativity



4. Let-P-R-commutativity



Experimental Results

As the recency bound increases, the coordination overhead and response time decrease.



- Synthesis of replicated objects that preserve integrity, convergence and recency, and minimize coordination
- Coordination conditions sufficient for these properties that are captured as commutativity conditions
- Reduced coordination minimization to classical graph optimization
- Coordination protocols that preserve these conditions

Commutativity Reasoning for Automated Distributed Coordination

> Mohsen Lesani University of California, Riverside






























































































Experiments







We execute 500 calls evenly distributed on the methods.

We increase the workload from 10 to 800 calls per second and measure the average response time over all the calls.










Experimental Results

As the recency bound increases, the coordination overhead and response time decrease.



Experimental Results

As the recency bound increases, the coordination overhead and response time decrease.



Experimental Results

As the recency bound increases, the coordination overhead and response time decrease.





 $\Sigma := \mathsf{let} \ rs := \mathsf{Set} \ \mathbb{N} \times \mathbb{N} \ \mathsf{in} \quad \triangleright \operatorname{Reservation: user identifier and movie identifier}$ let ms :=Set $\mathbb{N} \times \mathbb{N}$ in \triangleright Movie: movie identifier and available space $\langle rs, ms \rangle$ $\mathcal{I} := \lambda \langle rs, ms \rangle$. unique $(ms, \lambda \langle m, a \rangle, m) \land$ refintegrity $(rs, \lambda \langle u, m \rangle. m, ms, \lambda \langle m, a \rangle. m) \land$ rowIntegrity $(ms, \lambda \langle m, a \rangle, a > 0)$ $\mathsf{book}(\langle u, m \rangle) := 0 \ \lambda \langle rs, ms \rangle.$ $\langle \langle u, m \rangle \notin rs, \quad \langle rs \cup \langle u, m \rangle, \ \mathcal{U}_{\lambda \langle m', a \rangle, \langle m' = m, \langle m, a - 1 \rangle \rangle} \ ms \rangle, \quad \bot \rangle$ cancelBook($\langle u, m \rangle$) := 0 $\lambda \langle rs, ms \rangle$. $\langle \mathsf{True}, \langle rs \setminus \langle u, m \rangle, \mathcal{U}_{\lambda \langle m', a \rangle}, \langle m'=m, \langle m, a+1 \rangle \rangle} ms \rangle, \perp \rangle$ offScreen $(m) := 0 \lambda \langle rs, ms \rangle$. $\langle \mathsf{True}, \langle rs, ms \setminus \sigma_{\lambda \langle m', a \rangle \cdot m' = m} ms \rangle, \perp \rangle$ specialReserve($\langle m, n \rangle$) := 0 $\lambda \langle rs, ms \rangle$. $\langle n > 0, \langle rs, \mathcal{U}_{\lambda \langle m', a \rangle, \langle m' = m, \langle m, a - n \rangle \rangle} ms \rangle, \perp \rangle$ increaseSpace($\langle m, n \rangle$) := 0 $\lambda \langle rs, ms \rangle$. $\langle n > 0, \quad \langle rs, \ \mathcal{U}_{\lambda \langle m', a \rangle. \ \langle m' = m, \langle m, a + n \rangle \rangle} \ ms \rangle, \quad \bot \rangle$

Movie Booking use-case



Class MovieBooking

 $\Sigma :=$ let rs := Set $\mathbb{N} \times \mathbb{N}$ in \triangleright Reservation: user identifier and movie identifier let ms :=Set $\mathbb{N} \times \mathbb{N}$ in \triangleright Movie: movie identifier and available space $\langle rs, ms \rangle$ $\mathcal{I} := \lambda \langle rs, ms \rangle$. unique $(ms, \lambda \langle m, a \rangle, m) \land$ refintegrity $(rs, \lambda \langle u, m \rangle. m, ms, \lambda \langle m, a \rangle. m) \land$ rowIntegrity $(ms, \lambda \langle m, a \rangle, a > 0)$ $\mathsf{book}(\langle u, m \rangle) := 0 \ \lambda \langle rs, ms \rangle.$ $\langle \langle u, m \rangle \notin rs, \quad \langle rs \cup \langle u, m \rangle, \ \mathcal{U}_{\lambda \langle m', a \rangle, \langle m' = m, \langle m, a - 1 \rangle \rangle} \ ms \rangle, \quad \bot \rangle$ cancelBook($\langle u, m \rangle$) := 0 $\lambda \langle rs, ms \rangle$. $\langle \mathsf{True}, \langle rs \setminus \langle u, m \rangle, \mathcal{U}_{\lambda \langle m', a \rangle}, \langle m'=m, \langle m, a+1 \rangle \rangle} ms \rangle, \perp \rangle$ offScreen $(m) := 0 \lambda \langle rs, ms \rangle$. $\langle \mathsf{True}, \langle rs, ms \setminus \sigma_{\lambda \langle m', a \rangle \cdot m' = m} ms \rangle, \perp \rangle$ specialReserve($\langle m, n \rangle$) := 0 $\lambda \langle rs, ms \rangle$. $\langle n > 0, \langle rs, \mathcal{U}_{\lambda \langle m', a \rangle, \langle m' = m, \langle m, a - n \rangle \rangle} ms \rangle, \perp \rangle$ increaseSpace($\langle m, n \rangle$) := 0 $\lambda \langle rs, ms \rangle$. $\langle n > 0, \quad \langle rs, \ \mathcal{U}_{\lambda \langle m', a \rangle. \ \langle m' = m, \langle m, a + n \rangle \rangle} \ ms \rangle, \quad \bot \rangle$

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Reservation			Movie	
User ID: Integer	Movie ID: Integer	┝──►	Movie ID: Integer	Available Space: Integer

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 $\Sigma := \mathsf{let} \ rs := \mathsf{Set} \ \mathbb{N} \times \mathbb{N} \ \mathsf{in} \quad \triangleright \operatorname{Reservation: user identifier and movie identifier}$ let ms :=Set $\mathbb{N} \times \mathbb{N}$ in \triangleright Movie: movie identifier and available space $\langle rs, ms \rangle$ $\mathcal{I} := \lambda \langle rs, ms \rangle$. unique $(ms, \lambda \langle m, a \rangle, m) \land$ refintegrity $(rs, \lambda \langle u, m \rangle. m, ms, \lambda \langle m, a \rangle. m) \land$ rowIntegrity $(ms, \lambda \langle m, a \rangle, a > 0)$ $\mathsf{book}(\langle u, m \rangle) := 0 \ \lambda \langle rs, ms \rangle.$ $\langle \langle u, m \rangle \notin rs, \quad \langle rs \cup \langle u, m \rangle, \ \mathcal{U}_{\lambda \langle m', a \rangle, \langle m' = m, \langle m, a - 1 \rangle \rangle} \ ms \rangle, \quad \bot \rangle$ cancelBook($\langle u, m \rangle$) := 0 $\lambda \langle rs, ms \rangle$. $\langle \mathsf{True}, \langle rs \setminus \langle u, m \rangle, \mathcal{U}_{\lambda \langle m', a \rangle}, \langle m'=m, \langle m, a+1 \rangle \rangle} ms \rangle, \perp \rangle$ offScreen $(m) := 0 \lambda \langle rs, ms \rangle$. $\langle \mathsf{True}, \langle rs, ms \setminus \sigma_{\lambda \langle m', a \rangle \cdot m' = m} ms \rangle, \perp \rangle$ specialReserve($\langle m, n \rangle$) := 0 $\lambda \langle rs, ms \rangle$. $\langle n > 0, \langle rs, \mathcal{U}_{\lambda \langle m', a \rangle, \langle m' = m, \langle m, a - n \rangle \rangle} ms \rangle, \perp \rangle$ increaseSpace($\langle m, n \rangle$) := 0 $\lambda \langle rs, ms \rangle$. $\langle n > 0, \quad \langle rs, \ \mathcal{U}_{\lambda \langle m', a \rangle. \ \langle m' = m, \langle m, a + n \rangle \rangle} \ ms \rangle, \quad \bot \rangle$



request issued



request issued



request issued



request issued





request return







request return







request return







request return







request return

^جے synchronization





request return

^جے synchronization





request return

^جتر synchronization





request return

^Kしい synchronization






















$$\begin{array}{l} \mathsf{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \, ms \right) \rangle \\ \mathsf{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \, rs \right) \rangle \\ \mathsf{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \, ms \right) \rangle \end{array}$$

$$\begin{array}{l} \left(\begin{array}{ccc} \mathsf{querySpace}(m) = & \exists \lambda \langle rs, ms \rangle. \\ & \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \, ms \right) \right\rangle \\ \mathsf{queryReservations}(u) := & 4 \, \lambda \langle rs, ms \rangle. \\ & \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \, rs \right) \right\rangle \\ \mathsf{querySpaces}(u) := & 6 \, \lambda \, \langle rs, ms \rangle. \\ & \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \, (rs \, \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \, ms) \right\rangle \end{array}$$

$$\begin{array}{l} \mathsf{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \ \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. \ m' = m} \ ms \right) \rangle \\ \hline \mathsf{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \ \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. \ u' = u} \ rs \right) \rangle \\ \hline \mathsf{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. \ m = m'} \ ms \right) \rangle \end{array}$$

$$\begin{array}{l} \mathsf{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \ ms \right) \rangle \\ \mathsf{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \ rs \right) \rangle \\ \hline \mathsf{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \ ms \right) \rangle \end{array}$$

$$\begin{array}{l} \mathsf{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \, ms \right) \rangle \\ \mathsf{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \, rs \right) \rangle \\ \mathsf{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. \\ \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \, ms \right) \rangle \end{array}$$

Solution: $\Delta ms = 3$ $\Delta rs = 2$

$$\begin{array}{ll} \operatorname{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. & & \\ & \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \, ms \right) \rangle & & \\ \operatorname{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. & & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \, rs \right) \rangle & & \\ \operatorname{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. & & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \underbrace{\left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \, ms \right)} \right) & \\ \end{array}$$

$$\begin{array}{ll} \operatorname{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \, ms \right) \rangle & \\ \operatorname{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \, rs \right) \rangle & \\ \operatorname{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \, ms \right) \rangle \end{array}$$

$$\begin{array}{c}
\Gamma \vdash e \triangleright \delta, C \\
\Gamma \vdash e' \triangleright \delta', C' \\
\hline
\Gamma \vdash e \times e' \triangleright \delta \times \delta', C \wedge C'
\end{array}$$

$$\begin{array}{ll} \operatorname{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \, ms \right) \rangle & \\ \operatorname{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \, rs \right) \rangle & \\ \operatorname{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \, ms \right) \rangle \end{array}$$

$$\begin{array}{c}
\Gamma \vdash e \triangleright \delta, C \\
\Gamma \vdash e' \triangleright \delta', C' \\
\hline
\Gamma \vdash e \times e' \triangleright \delta \times \delta', C \wedge C'
\end{array}$$

$$\begin{array}{ll} \operatorname{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \, ms \right) \rangle & \\ \operatorname{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \, rs \right) \rangle & \\ \operatorname{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \, ms \right) \rangle \end{array}$$



$$\begin{array}{ll} \mathsf{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \, ms \right) \rangle & \\ \mathsf{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \, rs \right) \rangle & \\ \mathsf{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \, ms \right) \rangle \end{array}$$



$$\begin{array}{ll} \mathsf{querySpace}(m) := 3 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle m', a \rangle. \langle a \rangle} \left(\sigma_{\lambda \langle m', a \rangle. m' = m} \, ms \right) \rangle & \\ \mathsf{queryReservations}(u) := 4 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u', m \rangle. \langle m \rangle} \left(\sigma_{\lambda \langle u', m \rangle. u' = u} \, rs \right) \rangle & \\ \mathsf{querySpaces}(u) := 6 \ \lambda \langle rs, ms \rangle. & \\ & \langle ..., & ..., & \Pi_{\lambda \langle u, m, m', a \rangle} \langle m, a \rangle \left(rs \ \bowtie_{\lambda \langle u, m \rangle, \langle m', a \rangle. m = m'} \, ms \right) \rangle \end{array}$$

CPRODConstrains:
$$\Gamma \vdash e \triangleright \delta, C$$
 $\Delta ms \times \Delta rs \leq 6$ $\Gamma \vdash e' \triangleright \delta', C'$ $\Delta ms \leq 3$ $\overline{\Gamma \vdash e \times e' \triangleright \delta \times \delta', C \wedge C'}$ $\Delta rs \leq 4$



$$\begin{aligned} & \mathcal{P}(\sigma,c) & c(\sigma) = \langle -,\sigma',v \rangle \\ & c' = c \cdot \operatorname{call}(r) \\ & \operatorname{AllSComm}(c) \\ & \operatorname{InvSuff}(c') & \operatorname{LetPRComm}(c') \\ & \operatorname{call}' = \operatorname{call}[r \mapsto c'] \\ & \operatorname{xs}' = \begin{cases} \operatorname{xs}[n \mapsto (\operatorname{xs}(n) ::: r)] & \text{if } \operatorname{call}(r) = \operatorname{id} \\ & \operatorname{xs} & \text{else} \\ \\ & \operatorname{InBound}_{\langle \operatorname{orig, call}' \rangle}(\operatorname{xs}', n) \\ \hline & (h[n \mapsto (x \leftarrow c; s, \sigma, r)], t, \operatorname{xs}, \operatorname{orig}, \operatorname{call}) \\ & \xrightarrow{n, r, c} \\ & (h[n \mapsto (s[x \mapsto v], \sigma', r)], t, \operatorname{xs}', \operatorname{orig}, \operatorname{call}') \end{aligned}$$



$$\begin{aligned} & \mathcal{P}(\sigma,c) & c(\sigma) = \langle -,\sigma',v \rangle \\ & c' = c \cdot \operatorname{call}(r) \\ & \mathsf{AllSComm}(c) \\ & \mathsf{InvSuff}(c') & \mathsf{LetPRComm}(c') \\ & \mathsf{call}' = \mathsf{call}[r \mapsto c'] \\ & \mathsf{xs}' = \begin{cases} \mathsf{xs}[n \mapsto (\mathsf{xs}(n) ::: r)] & \text{if } \mathsf{call}(r) = \mathsf{id} \\ \mathsf{xs} & \mathsf{else} \\ \\ & \mathsf{InBound}_{\langle \mathsf{orig}, \mathsf{call}' \rangle}(\mathsf{xs}', n) \\ \hline & (h[n \mapsto (x \leftarrow c; s, \sigma, r)], t, \mathsf{xs}, \mathsf{orig}, \mathsf{call}) \\ & \xrightarrow{n, r, c} \\ & (h[n \mapsto (s[x \mapsto v], \sigma', r)], t, \mathsf{xs}', \mathsf{orig}, \mathsf{call}') \end{aligned}$$



$$\begin{aligned} & \mathcal{P}(\sigma,c) & c(\sigma) = \langle -,\sigma',v \rangle \\ & c' = c \cdot \operatorname{call}(r) \\ & \operatorname{AllSComm}(c) \\ & \operatorname{InvSuff}(c') & \operatorname{LetPRComm}(c') \\ & \operatorname{call}' = \operatorname{call}[r \mapsto c'] \\ & \operatorname{xs}' = \begin{cases} \operatorname{xs}[n \mapsto (\operatorname{xs}(n) ::: r)] & \text{if } \operatorname{call}(r) = \operatorname{id} \\ & \operatorname{xs} & \operatorname{else} \end{cases} \\ & \underbrace{\operatorname{InBound}_{\langle \operatorname{orig, call}' \rangle}(\operatorname{xs}', n)} \\ & (h[n \mapsto (x \leftarrow c; s, \sigma, r)], t, \operatorname{xs}, \operatorname{orig}, \operatorname{call}) \\ & \xrightarrow{n, r, c} \\ & (h[n \mapsto (s[x \mapsto v], \sigma', r)], t, \operatorname{xs}', \operatorname{orig}, \operatorname{call}') \end{aligned}$$



CALLLOCAL

$$\mathcal{P}(\sigma, c) \qquad c(\sigma) = \langle -, \sigma', v \rangle$$

$$c' = c \cdot \operatorname{call}(r)$$
AllSComm(c)

$$\operatorname{InvSuff}(c') \qquad \operatorname{LetPRComm}(c')$$

$$\operatorname{call'} = \operatorname{call}[r \mapsto c']$$

$$\operatorname{xs'} = \begin{cases} \operatorname{xs}[n \mapsto (\operatorname{xs}(n) ::: r)] & \text{if call}(r) = \operatorname{id} \\ \operatorname{xs} & \text{else} \end{cases}$$

$$\operatorname{InBound}_{\langle \operatorname{orig, call'} \rangle}(\operatorname{xs'}, n)$$

$$(h[n \mapsto (x \leftarrow c; s, \sigma, r)], t, \operatorname{xs}, \operatorname{orig}, \operatorname{call})$$

$$\xrightarrow{n, r, c} \\ (h[n \mapsto (s[x \mapsto v], \sigma', r)], t, \operatorname{xs'}, \operatorname{orig}, \operatorname{call'})$$



CALLLOCAL

$$\mathcal{P}(\sigma, c) \qquad c(\sigma) = \langle -, \sigma', v \rangle$$

$$c' = c \cdot \operatorname{call}(r)$$
AllSComm(c)

$$\operatorname{InvSuff}(c') \qquad \operatorname{LetPRComm}(c')$$

$$\operatorname{call'} = \operatorname{call}[r \mapsto c']$$

$$\operatorname{xs'} = \begin{cases} \operatorname{xs}[n \mapsto (\operatorname{xs}(n) ::: r)] & \text{if call}(r) = \operatorname{id} \\ \operatorname{xs} & \text{else} \end{cases}$$

$$\operatorname{InBound}_{\langle \operatorname{orig, call'} \rangle}(\operatorname{xs'}, n)$$

$$(h[n \mapsto (x \leftarrow c; s, \sigma, r)], t, \operatorname{xs}, \operatorname{orig}, \operatorname{call})$$

$$\xrightarrow{n, r, c}$$

$$(h[n \mapsto (s[x \mapsto v], \sigma', r)], t, \operatorname{xs'}, \operatorname{orig}, \operatorname{call'})$$



$$\begin{aligned} & \mathcal{P}(\sigma,c) & c(\sigma) = \langle_{-},\sigma',v\rangle \\ & c' = c \cdot \operatorname{call}(r) \\ & \operatorname{AllSComm}(c) \\ & \operatorname{InvSuff}(c') & \operatorname{LetPRComm}(c') \\ & \operatorname{call}' = \operatorname{call}[r \mapsto c'] \\ & \operatorname{call}' = \operatorname{call}[r \mapsto c'] \\ & \operatorname{rs}' = \begin{cases} \operatorname{xs}[n \mapsto (\operatorname{xs}(n) ::: r)] & \text{if } \operatorname{call}(r) = \operatorname{id} \\ & \operatorname{xs} & \operatorname{else} \\ & \operatorname{InBound}_{\langle \operatorname{orig, call}' \rangle}(\operatorname{xs}', n) \\ \hline & (h[n \mapsto (x \leftarrow c; s, \sigma, r)], t, \operatorname{xs}, \operatorname{orig}, \operatorname{call}) \\ & \xrightarrow{n, r, c} \\ & (h[n \mapsto (s[x \mapsto v], \sigma', r)], t, \operatorname{xs}', \operatorname{orig}, \operatorname{call}') \end{aligned}$$



CALLLOCAL

$$\mathcal{P}(\sigma, c) \qquad c(\sigma) = \langle -, \sigma', v \rangle$$

$$c' = c \cdot \text{call}(r)$$
AllSComm(c)
InvSuff(c') LetPRComm(c')
call' = call[r \mapsto c']
xs' = \begin{cases} xs[n \mapsto (xs(n) ::: r)] & \text{if call}(r) = \text{id} \\ xs & \text{else} \end{cases}
$$\frac{\ln\text{Bound}_{\langle \text{orig, call'} \rangle}(xs', n)$$

$$(h[n \mapsto (x \leftarrow c; s, \sigma, r)], t, xs, \text{orig, call})$$

$$(h[n \mapsto (s[x \mapsto v], \sigma', r)], t, xs', \text{orig, call'})$$



$$\begin{aligned} & \mathcal{P}(\sigma,c) & c(\sigma) = \langle -,\sigma',v \rangle \\ & c' = c \cdot \operatorname{call}(r) \\ & \operatorname{AllSComm}(c) \\ & \operatorname{InvSuff}(c') & \operatorname{LetPRComm}(c') \\ & \operatorname{call}' = \operatorname{call}[r \mapsto c'] \\ & \operatorname{xs}' = \begin{cases} \operatorname{xs}[n \mapsto (\operatorname{xs}(n) ::: r)] & \text{if } \operatorname{call}(r) = \operatorname{id} \\ & \operatorname{xs} & \text{else} \\ \\ & \operatorname{InBound}_{\langle \operatorname{orig, call}' \rangle}(\operatorname{xs}', n) \\ \hline & (h[n \mapsto (x \leftarrow c; s, \sigma, r)], t, \operatorname{xs}, \operatorname{orig}, \operatorname{call}) \\ & \xrightarrow{n, r, c} \\ & (h[n \mapsto (s[x \mapsto v], \sigma', r)], t, \operatorname{xs}', \operatorname{orig}, \operatorname{call}') \end{aligned}$$





























State of the art

	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	✓	✓	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	~	~	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	~	×	~
Hampa	~	~	✓	~	~



State of the art

	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	~	✓	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	✓	~	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	~	×	~
Hampa	~	~	✓	✓	~


	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	✓	✓	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	~	~	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	~	×	~
Hampa	~	~	1	~	~



	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	✓	~	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	✓	~	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	✓	×	~
Hampa	~	~	✓	✓	✓



	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	✓	✓	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	~	~	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	~	×	~
Hampa	~	~	1	~	~



	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	✓	✓	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	✓	✓	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	~	×	~
Hampa	~	~	✓	✓	~



	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	✓	✓	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	~	~	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	~	×	~
Hampa	~	~	1	~	~



	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	✓	✓	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	✓	~	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	~	×	~
Hampa	~	~	✓	~	~



	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	✓	✓	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	~	~	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	~	×	~
Hampa	~	~	✓	~	~



	Convergence	Integrity	Recency	Synchronization avoidance	Communication avoidance
Strong consistency	✓	✓	✓	×	×
Eventual consistency / CRDT	✓	×	×	✓	×
Sieve, Indigo, CISE, Hamsaz, Soteria	✓	✓	×	~	×
TACT, TRAPP, FRACT, PBS	~	×	~	×	✓
Hampa	~	~	~	~	~



Reservation		Movie	
User ID: Integer Movie ID: Integer	├── ►	Movie ID: Integer	Available Space: Integer



Reservation		Movie	
User ID: Integer Movie ID: Integer	├── ►	Movie ID: Integer	Available Space: Integer

Reservation		Movie
User ID: Integer Movie ID: Integer	┝──┝	Movie ID: Integer Available Space: Integer

Reservation		Movie	
User ID: Integer Movie ID: Integer	├── ►	Movie ID: Integer	Available Space: Integer









Input

Output



Input

Output



Input

Output



Input

Output



Input

Output



Input

Output



Input

Output



Input

Output

offScreen method



Input

Output

offScreen method



Input

Output

specialReserve method



Input

Output

specialReserve method



Input

Output

specialReserve method



Input

Output

increaseSpace method



Input

Output

increaseSpace method



Input

Output

increaseSpace method



Input

Output

Facilitating the consistency choices

- Require user-specified choices or annotations
- Crucially dependent on causal consistency as the weakest notion